How does a twisted beliefs shock affect the macroeconomy?*

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Abstract

I study the role of "shattered" or "twisted" beliefs combined with Bayesian learning in a standard equilibrium business cycle framework. By adapting ideas from Cogley and Sargent (2008b) to the general equilibrium setting, I am able to study how a prior belief arising from the Great Depression may have influenced the macroeconomy during the last 75 years. In the model, households hold twisted beliefs concerning the likelihood and persistence of recession and boom states, beliefs which are only gradually unwound during subsequent years. Even though the driving stochastic process for technology is unchanged over the entire period, the nature of macroeconomic performance is altered considerably for many decades before eventually converging to the rational expectations equilibrium. This provides some evidence of the lingering effects of beliefs-twisting events on the behavior of macroeconomic variables.

Keywords: Bayesian learning, business cycles
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1 Introduction

1.1 Motivation

In a provocative analysis of asset pricing puzzles, Cecchetti, Lam, and Mark (2000) showed that if households' beliefs about the driving stochastic process are "twisted" in a particular way, an otherwise standard asset pricing model could be consistent with asset pricing facts, in particular, the equity premium studied by Mehra and Prescott (1985). In Cecchetti et al. (2000), households do not update their beliefs over time. Cogley and Sargent (2008b) extended the analysis of Cecchetti et al. (2000) adding learning in the following way. They suggested that the Great Depression was a "beliefs twisting event" citing Friedman and Schwartz (1963). Friedman and Schwartz (1963) suggested that the Great Depression "shattered" beliefs in the future of capitalism. Cogley and Sargent (2008b) captured this shattering of beliefs as a particular representation of a transition probability matrix in a Bayesian learning version of Mehra-Prescott. They found that they could match the asset pricing facts as Cecchetti et al. (2000) did, but that the equilibrium dynamics would eventually converge to rational expectations and thus that, in particular, the equity premium would converge to the negligible rational expectations value. However, this process took decades, according to their analysis. They suggested that this might provide an interesting part of the explanation of the equity premium puzzle in the postwar U.S. data.

In this paper I study the twisted beliefs idea of Cecchetti et al. (2000) and Cogley and Sargent (2008b) in a standard dynamic stochastic general equilibrium macroeconomic context. If beliefs were shattered as described by Friedman and Schwartz (1963), then one would expect the behavior of the private sector to change and that this should affect all aspects of the evolution of the economy. Further, the slow convergence described by Cogley and Sargent (2008b) may suggest that these effects would be very persistent. The goal of this paper is to investigate these ideas.

1.2 What I do

The core idea is to consider a standard equilibrium business cycle framework under twisted beliefs and Bayesian learning. In the paper, productivity follows an observable exogenous stochastic regime-switching process. In contrast to the standard model, I assume that households have subjective beliefs about the distribution of productivity that may not coincide with the true data generating process. Agents learn by starting with initial beliefs and updating them according to Bayes law. When existing beliefs are "shattered" agents have to learn beginning with their new priors. Without twisted beliefs, this economy would deliver the equilibrium business cycle properties as described by Prescott (1986) and Auroba, Fernandez-Villaverde, and Rubio-Ramirez (2006). I study the effects of a one-time "shattering" of beliefs on this economy similar to the one studied by Cogley and Sargent (2008b) in the Mehra-Prescott partial equilibrium asset pricing problem. I stress that, while I am studying a particular beliefs-twisting event, the core idea would apply equally well to any such event. I solve the model by value function iteration. I compare how the behavior of the economy with twisted beliefs and Bayesian learning differs from the rational expectations version.

1.3 Main findings

The main findings indicate that for a sufficiently large shock to the beliefs of the agents, the macroeconomic impact can be quantitatively important. In addition, these effects can be very persistent, taking many decades to play out through the macroeconomy. This is because it takes a long time to correct the pessimistic beliefs induced by the depression event through the observation of macroeconomic data. This suggests that belief-twisting events may have long-lasting impacts on the macroeconomy through a channel not studied in the previous literature. Many writers since the 1930s have argued informally that the Great Depression created a "depression generation" that behaved in a way that affected the macroeconomy for decades after the depression ended.[1] This conjecture is borne out by the quantitative analysis in this paper.

1.4 Recent literature

This paper is related to an emerging literature on the effects of learning on the economy in the standard real business cycle framework.

Williams (2003), Carceles-Poveda and Giannitsarou (2007), Eusepi and Preston (2008), and Huang, Liu, and Zha (2008) all use an adaptive learning approach in a standard RBC model. They consider specifications in which agents learn about reduced-form equilibrium laws of motion. Williams (2003) and Carceles-Poveda and Giannitsarou (2007) consider learning in which only one-period-ahead forecasts matter for household and firm behavior. They find that learning dynamics do not have quantitatively impor-

[1] For example, Danthine and Donaldson (1999) note:

"Yet, it is not unreasonable to think, for example, that the experience of the Great Depression continues to have a significant influence on the behaviour of those who experienced it directly or indirectly, even though it has not recurred in sixty-five years" p. 608.

tant effect on the properties of business cycles or asset pricing behavior. Eusepi and Preston (2008) extend this model with multi-period-ahead forecasts and show that the quantitative effects of adaptive learning in dynamic general equilibrium models can be significant. Huang, Liu, and Zha (2008) reach a similar conclusion for the model with self-confirming equilibria.

The two-state regime switching representation of business cycles model started with Hamilton (1989). Most recently it was used in papers of Cagetti, Hansen, Sargent, and Williams (2002), Nieuwerburgh and Veldkamp (2006), and Bullard and Singh (2008). These papers assume imperfect information about current state so that agents have to solve a signal extraction problem.

The influence of beliefs on the economy has been studied extensively in the asset pricing literature. Rietz (1988), Abel (2002), and Barro (2006), among others, study the effect of a non-zero probability of a "disaster" state on agents' subjective expectations in the model without learning. Kurz, Jin, and Motolese (2005) employ Kurz's (1994) concept of rational beliefs and show how subjective beliefs and learning affect asset pricing. Guidolin (2006) studies the impact of initial pessimism on equity premium in the model with rational learning. Weitzman (2007) shows that in an asset pricing model with Bayesian updating of unknown structural parameters, subjective prior beliefs play important and persistent role in determination of asset prices.

Jaimovich and Rebelo (2006, 2007) study the business cycle response to mistakenly optimistic beliefs about productivity in the neoclassical growth model with variable capital utilization, investment adjustment costs, and preferences implying a weak short-run wealth effect on the labor supply. They show that incorrect beliefs can generate fluctuations in the economy but that the quantitative effect on volatility is small.

1.5 Organization

The next section describes the environment. Section 3 presents results for baseline case. Section 4 addresses some alternative specifications and explores the robustness of the findings. The last section concludes and suggests directions for future research.

2 Environment

I consider a standard equilibrium business cycle model. The core idea is to use a completely standard macroeconomic model, in which the only addition is to twist beliefs as in Cogley and Sargent (2008b) and to allow agents to learn via Bayesian methods. Ultimately, the households will again learn the rational expectations equilibrium following the shock to beliefs. Once this convergence occurs, the economy will behave exactly as the standard results suggest. During the transition, however, the economy may depart from the rational expectations norm, and I will present results illustrating the nature of this departure.

The stochastic process for productivity is exogenous and does not depend on any action taken by agents. Therefore, there is no incentive for "active" learning with agents taking action that would allow them to understand the stochastic process better.

2.1 Preferences and endowments

The household portion of the model is entirely standard. The representative household has preferences over stochastic stream of consumption, c and leisure, l, with utility at time t given by

$$U_0 = \hat{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \tag{1}$$

where β is the discount factor and $\hat{E}_t$ is the subjective expectation operator. Rational expectations can be considered as a special case where the subjective probability distribution coincides with the true data generating process. The instantaneous utility function, u, is given by

$$u(c_t, l_t) = \begin{cases} \frac{[c_t^{\theta} l_t^{1-\theta}]^{1-\tau}}{1-\tau}, & \text{if } \tau \neq 1, \\ \theta \log(c_t) + (1-\theta)\log(l_t), & \text{if } \tau = 1. \end{cases} \tag{2}$$

The parameter τ control intertemporal elasticity of substitution of consumption–leisure bundles, and θ governs intratemporal elasticity of substitution between consumption and leisure.

In each period, the representative household has one unit of time which it allocates between labor and leisure. The household is also endowed with initial stock of capital k_0, which can be augmented through investment, x_t. The law of motion for the capital is then given by

$$k_{t+1} = (1-\delta)k_t + x_t, \tag{3}$$

where δ is the net depreciation rate of the existing capital stock.

2.2 Technology

The technology is also standard. A representative firm operates the stochastic production technology to produce output, y_t, with capital, k_t, and labor, l_t,

$$y_t = A_t f(k_t, l_t), \tag{4}$$

with the constant return to scale, increasing, concave, twice continuously differentiable production function f,

$$f(k_t, n_t) = k_t^{\alpha} l_t^{1-\alpha}. \tag{5}$$

The variable A_t follows a stochastic process modeled as $A_t = e^{z_t}$ with z_t representing the level of technology relative to a balanced growth path.

The level of technology follows a two–state Markov switching process, $z_t \in \{z_L, z_H\}$, modeled as

$$z_t = z_H S_t + z_L(1 - S_t), \qquad z_H > z_L,$$

with $S_t = 1$ denoting an "expansion" state and $S_t = 0$ being a "recession" state. States follow a Markov switching process with the transition probability matrix

$$\Pi = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix},$$

where $p = Prob(S_{t+1} = 1 | S_t = 1)$ and $q = Prob(S_{t+1} = 0 | S_t = 0)$.

2.3 Information and beliefs

In the model I allow for agents not having the full knowledge of the data generating process for productivity. I assume agents know productivity is governed by two–state Markov regime switching process, know the growth rate in each state (i.e. know z_H and z_L) but do not know the probability transition matrix Π. Agents are Bayesian learners: they start with prior beliefs, Π_0, summarizing their perception of the economy, and update them as they observe the actual states.

Assume agents' prior beliefs are beta distributed with

$$\begin{aligned} p &\sim Beta(u_{00}, u_{01}), \\ q &\sim Beta(u_{11}, u_{10}). \end{aligned}$$

If p and q are independently distributed then the joint prior distribution of $\pi_0(p, q)$ is

$$\pi_0(p, q) \propto p^{u_{00}}(1-p)^{u_{01}} q^{u_{11}}(1-q)^{u_{10}}. \tag{6}$$

Agents update their beliefs according to Bayes' law and after observing the actual sequence of states, S^t, they are

$$\begin{aligned} f(p,q|S^t) &= f(S^t)^{-1} f(p,q,S^t) = f(S^t)^{-1} f(S^t|p,q)\pi(p,q) \\ &\propto L(p,q|S^t)\pi(p,q), \end{aligned}$$

where $f(S^t|p,q) = L(p,q|S^t)$ is the likelihood function,

$$L(p,q|S^t) \propto p^{m_{00}}(1-p)^{m_{01}} q^{m_{11}} (1-q)^{m_{10}}. \tag{7}$$

Here, m_{ij} denotes number of times the process transitioned from state i to state j in the sequence S_t.

Then, the posterior distribution of transition probability matrix, Π_t, is given by

$$f(p,q|S^t) \propto p^{u_{00}+m_{00}-1}(1-p)^{u_{01}+m_{01}-1} q^{u_{11}+m_{11}-1}(1-q)^{u_{10}+m_{10}-1},$$

implying beta posterior distributions of p and q:

$$\begin{aligned} p_t &\sim Beta(u_{00}+m_{00,t}, u_{01}+m_{01,t}), \\ q_t &\sim Beta(u_{11}+m_{11,t}, u_{10}+m_{10,t}). \end{aligned}$$

According to the Bayes' consistency theorem, the posterior distribution will converge to the data generating process.

The distribution of beliefs and the updating procedure is summarized by counters $n_{i,j} = u_{i,j} + m_{i,j}$, which are sufficient statistics for the beta distribution. Under the distributional assumptions, the expected probabilities of transition, p^e and q^e, are

$$p^e = E(p) = \frac{n_{11}}{n_{11}+n_{12}}, \tag{8}$$

$$q^e = E(q) = \frac{n_{22}}{n_{11}+n_{12}}, \tag{9}$$

and their updates are summarized by the evolution of the counters.

If $\Pi_t(S^t)$ is the posterior probability given prior beliefs and sequence of states S^t, it can be represented with $n_t = \{n_{00t}, n_{01t}, n_{10t}, n_{11t}\}$.[2] We can consider n_t as a state variable as together with S_t it describes the current state of the beliefs in the economy. The transition equation for n_t is as follows

$$\begin{aligned} n_{t+1} &= n_t + \{1,0,0,0\}, \quad \text{if } S_t = 0, S_{t+1} = 0, \\ n_{t+1} &= n_t + \{0,1,0,0\}, \quad \text{if } S_t = 0, S_{t+1} = 1, \\ n_{t+1} &= n_t + \{0,0,1,0\}, \quad \text{if } S_t = 1, S_{t+1} = 0, \\ n_{t+1} &= n_t + \{0,0,0,1\}, \quad \text{if } S_t = 1, S_{t+1} = 1, \end{aligned}$$

[2] This representation of beliefs is used in Cogley and Sargent (2008a, 2008b).

where $n_{t+1} = \{n_{00,t+1}, n_{01,t+1}, n_{10,t+1}, n_{11,t+1}\}$.

In such a formulation, the sufficient statistic, n, governs both the beliefs about transition matrix and the precision of these beliefs. To see this, consider two probability transition matrices represented by $n^l = \{4, 1, 1, 4\}$ and $n^k = \{40, 10, 10, 40\}$. They feature the same probabilities of expansion and recession, but after observing a series of states, beliefs given by the n^k vector will be less influenced by the incoming data relative to the one given by n^l. In this sense, the beliefs represented by n^k are more dogmatic than those represented by n^l. In the analysis below, much will depend on the moment at which beliefs are shattered and the counters that are used by the representative household to describe the new beliefs following the beliefs-twisting event.

In this paper, we consider the case of adaptive learning with agents treating their current state of beliefs about distribution of stochastic process as the true one. They do not take into account future updating of their beliefs.[3]

2.4 Timing

At the beginning of the period both the household and the firm observe the level of technology. The household updates its beliefs about distribution governing states and chooses the level of investment, consumption, and leisure; the firm chooses amount of capital to rent and labor to hire from household. The interest rate and the wage clear both markets.

2.5 Firm's problem

Each period, the representative firm chooses how much capital to rent from household and how much labor to hire to maximize profits,

$$\max_{k_t, l_t} \{e^{z_t} f(k_t, l_t) - w_t l_t - r_t k_t\}, \qquad \forall t. \tag{10}$$

The first order conditions for the firm problem determines the prices of factor of productions,

$$r_t = e^{z_t} f_k(k_t, l_t), \tag{11}$$

$$w_t = e^{z_t} f_l(k_t, l_t). \tag{12}$$

[3] Cogley and Sargent (2008a) show that the consumption and investment choices under fully Bayesian (i.e. internalizing future updating) and adaptive learning behavior are very similar for low values of the coefficient of relative risk aversion.

2.6 Household's problem

The representative household chooses stream of consumption, leisure, and investment $\{c_t, l_t, x_t\}_{t=0}^{\infty}$ to maximize its expected lifetime utility (1) subject to a sequence of budget constraints

$$c_t + x_t \leq w_t l_t + r_t k_t, \tag{13}$$

where the wage rate and the interest rate are taken by household as given.

2.7 Planner's problem and recursive competitive equilibrium

The benevolent social planner chooses the sequences for consumption, labor supply, and the capital stock to maximize household's utility in (1) subject to the prior beliefs, the initial level of capital stock, technology and the sequence of resource constraints,[4]

$$\begin{aligned} \max_{c_t, l_t, k_{t+1}} \quad & \hat{E}_0 \sum_t \beta^t u(c_t, l_t) \\ \text{s.t.} \quad & c_t + k_{t+1} = e^{z_t} f(k_t, l_t) \\ & k_0 > 0 \end{aligned}$$

The planner's problem can be cast in a recursive fashion. The state variables in dynamic programming formulation are the state of the economy, and the capital stock $\vartheta_t = (s_t, k_t)$. The dynamic programming problem can be written in terms of the Bellman equation

$$\begin{aligned} v(s,k) = \; & \max\nolimits_{c,x,l} \{u(c,l) + \beta \hat{E}[v(s',k')|s]\} \\ \text{s.t.} \quad & c + x \leq r(s,k)\, k + w(s,k)\, l, \\ & k' = (1-\delta)k + x, \\ & c \geq 0, \; 0 \leq l \leq 1, \; k_0. \end{aligned} \tag{14}$$

Conditional on agents' perception of stochastic process, the recursive competitive equilibrium consists of value function v; policy function $c(\vartheta)$, $l(\vartheta)$, and $x(\vartheta)$ for household, and price functions, $w(\vartheta)$ and $r(\vartheta)$, such that these functions are consistent with (a) the representative household's problem; (b) the firm's maximization problem; and (c) the resource constraint, $c + x = y$, $\forall (s,k)$.

2.8 Characterization of the equilibrium

The social planner's problem can be restated as choosing k' and l in

$$\begin{aligned} v(s,k) \quad &= \quad \max_{k',l} \{u(r\,k + w\,l + (1-\delta)k - k', l) + \beta \hat{E}[v(s',k')|s]\} \\ \text{s.t.} \quad & \quad\quad c \geq 0, \; 0 \leq l \leq 1, \; k_0. \end{aligned} \tag{15}$$

[4] I assume social planner and the representative household have the same prior beliefs.

The first order condition for consumption is

$$u_c(c,l) - \beta \hat{E}[v_k(s',k')|s] = 0,$$

and for labor

$$-u_l(c,l) + \beta \hat{E}[v_k(s',k')w(s,k)|s] = 0.$$

Using envelope theorem, v_k can be written as

$$v_k(s,k) = u_c(c,l)[r(s,k) + (1-\delta)].$$

The equilibrium in this economy is, therefore, described by the optimality conditions:

$$\begin{aligned}
u_c(c,l) &= \beta \hat{E}[u_c(c',l')(r' + 1 - \delta)|s], && (16)\\
u_n(c,l) &= u_c(c,l)w, && (17)\\
r &= e^z f_k(k,l), && (18)\\
w &= e^z f_l(k,l), && (19)\\
c + k' &= e^z f_k(k,l) + (1-\delta)k. && (20)
\end{aligned}$$

The time-varying expectations $\hat{E}$ are taken with respect to probabilities which are updated as described earlier.

2.9 Twisted beliefs

The optimal decision depends on expectations of future productivity. Under our assumptions, expectations are changing over time and at any date t depend on initial beliefs and the actual sequence of observed states S^t. The true data generating process for productivity, z_t, is exogenous. The posterior beliefs, however, reflect subjective perceptions embodied in the prior along with agents' observations of the stochastic process driving the evolution of the economy.

In this paper, I am interested in studying the effects of "shattered" beliefs—events that change households' perceptions about the stochastic process driving economy. Since households in the paper are Bayesian learners, eventually they will learn the true process. However, in the meantime, the beliefs-twisting event has clear effects on actual household behavior. One logical choice for the twist in beliefs is the Great Depression as studied by Cogley and Sargent (2008b).[5] However, the Great Depression does not have to be the only possible beliefs-twisting event. The spirit of this paper is to find a generic

[5] According to Friedman and Schwartz (1963) the Great Depression of 1930s persistently changed the perception about the nature of processes governing economy:

Table 1: Standard deviations.

Variable	RE Model	Aruoba et al. (2006)
Output	1.2314	1.2418
Consumption	0.4619	0.4335
Hours	0.5929	0.5714
Investment	3.6546	3.6005
Capital	0.2502	0.2490

Note: Average (400 simulations) percentage deviations from Hodrick-Prescott trend.

description of the behavior following any such event. There may be many other cases, especially outside the post-war G7 economies.

2.10 Solution method

To find the optimal decision rule in the social planner problem, I use value function iteration on a grid. With capital being the only continuous variable, it never leaves the grid making the solution exact. Since I consider the adaptive learning so that subjective beliefs are not part of the state vector in the dynamic programming problem, the decision rule is computed on a grid of possible subjective beliefs. Then, as the beliefs are updated the decision rule corresponding to these beliefs is used.

The changing decision rules introduces new the source of variability in the macro variables. The fluctuations in macro variables are now the result of both stochastic productivity and the changes in the decision rule.

2.11 Calibration

I calibrate the model at a quarterly frequency and follow parameterization of Aruoba, et al. (2006). The utility function parameters are set to $\beta = 0.9896$, $\tau = 2$, and $\theta = 0.357$ implying steady state values for annual interest rate of 4% and labor supply of 31% of available time. The technology parameters are set to $\alpha = 0.4$ and $\delta = 0.0196$.

I chose the parameters of the stochastic process for productivity to match the stochastic characteristics of the AR(1) process for Solow residual of the U.S. economy in $z_t = \rho z_{t-1} + \varepsilon_t$ with $\rho = 0.95$ and $\sigma_\varepsilon = 0.007$. For the baseline calibration, I fol-

"The contraction after 1929 shattered beliefs in a 'new era'... . The contraction instilled instead an exaggerated fear of continued economic instability, of the danger of stagnation, of the recurrent unemployment." (p.673)

Table 2: Baseline priors.

Process	Counters				Probabilities			
	u_{11}	u_{01}	u_{10}	u_{00}	p	q	$Pr(S_t = 1)$	$Pr(S_t = 0)$
True Process	39	1	1	39	0.975	0.975	0.50	0.50
Baseline Prior	2	2	2	28	0.500	0.933	0.12	0.88

low Bullard and Singh (2007) and Van Nieuwerburgh and Veldkamp (2006) assuming symmetric probabilities $p = q = 0.975$ and symmetric regimes $z_H = -z_L = 0.0225$.[6]

The comparison of standard deviations of key endogenous variables for a standard equilibrium business cycle model and the paper's two-state regime switching representation is in Table 1. The table shows that the baseline model–without twisted beliefs–does not differ significantly from Aruoba et al. (2006).

3 Twisted beliefs in the baseline model

The purpose of this paper is to study the case where the initial beliefs about the data generating process for productivity disagree with the true transition probabilities and agents learn and update their beliefs as time passes. The degree of disagreement will determine how far away the agents' initial perception of the economy is from the truth.

For the baseline calibration I simulate the model with priors that represent pessimistic "twisted" beliefs represented in Table 2. I endow agents with twisted beliefs that differ substantially from the data generating process for productivity in three dimensions. First, agents see productivity as governed by an asymmetric process with expansions lasting on average 2 quarters and recessions lasting on average almost 4 years. This is in stark contrast from the true data generating process according to which both states last on average 10 years. Second, given the true data generating process, agents have a relatively uninformative prior concerning expansions. Lastly, agents are underestimating the persistence of both states. These priors are consistent with agents taking NBER dates on recessions and expansions for the period of 1929:2–1933:3. In particular, this is what agents would use based on counters taking the beginning of "new era" as 1929:2 at the end of 1933. This is just a baseline case—I will study different sets of priors and an alternative data generating process for productivity in the following section.

[6] Bullard and Singh (2007) set $z_H = -z_L = 0.0035$ while Van Nieuwerburgh and Veldkamp (2006) use $z_H = -z_L = 0.032$. See the Appendix for AR(1) representation of the productivity process z_t.

In following subsections I compare how the evolution of the economy populated by Bayesian agents with twisted beliefs differs from the economy with rational expectations' agents.

For each simulation, a sequence of 400 productivity shocks is drawn from the true distribution. When forming expectations, rational expectations agents use the true transition probability matrix Π. In contrast, Bayesian agents start with initial priors Π_0 and update their beliefs with realizations of the stochastic process. The stock of capital in the economy, k_0, is initialized at the deterministic steady state level and agents make their optimal decisions according to equations (16)–(20).[7] I compare how decisions concerning consumption, investment, labor supply, and the evolution of other macroeconomic variables differ under the assumption of Bayesian learning as compared to the assumption of full information, rational expectations.

3.1 Evolution of variables

I compare percentage deviations from the steady state under rational expectations and under Bayesian learning. Figures 1 and 2 portray the evolution of macroeconomic variables for two individual simulations. These two figures differ with respect to the realization of the draw of stochastic productivity and, accordingly, the speed of convergence of learning.

Figure 1 presents a simulation with slow convergence of probabilities and, as a result, with slow convergence of decision rules under learning to decision rules under rational expectations. As agents observe a sequence of draws mostly from the recession state, they cannot update their pessimistically twisted beliefs about perisistence of expansion state. Agents expect expansions to be short, quickly followed by a recession and they choose to accumulate more capital comparing to the full information case. Moreover, agents over–estimate the persistence of the recession state distorting decisions even more.

In contrast, Figure 2 presents a simulation with relatively fast convergence to rational expectations. As agents observe a long sequence of draws from the expansion state, they can update their mistaken beliefs more readily. However, agents' initial behavior is still distinctively different from the full information case. As they expect the recession state to occur more often than the true probability indicates, once they are in the

[7] One may want to consider the situation where initial capital is not at steady state. Starting away from steady state can be considered but since I consider deviations of rational expectations from learning such an approach would generate the same results.

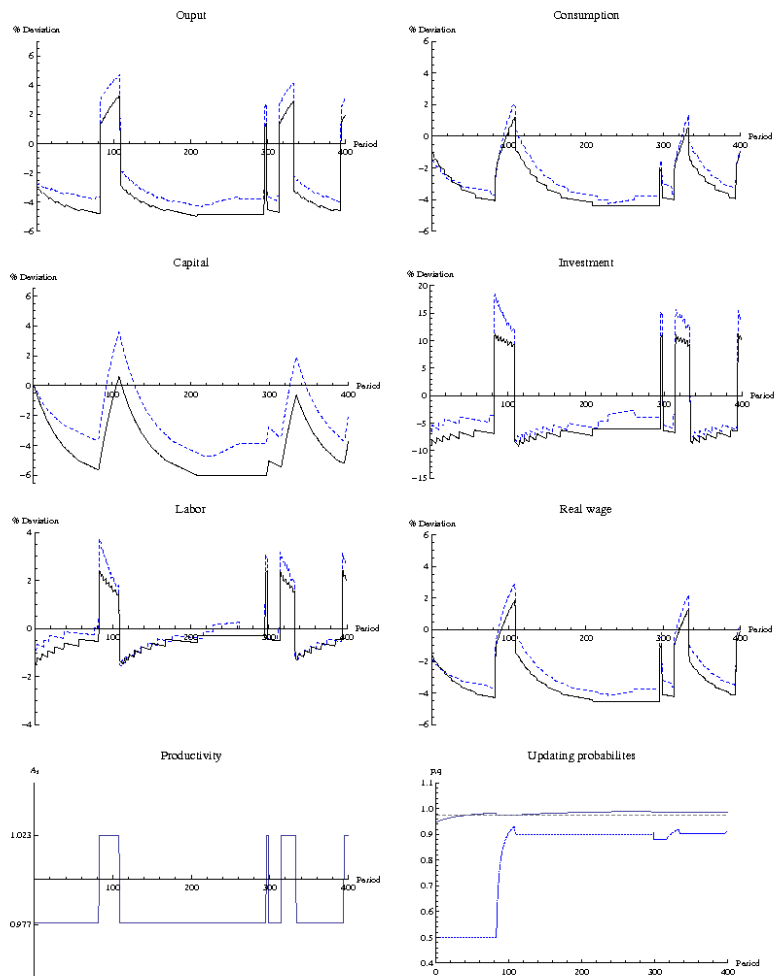

Figure 1: Individual simulation: Slow convergence. Dashed line represents deviations from steady state under learning; solid line is deviations from steady state under rational expectations.

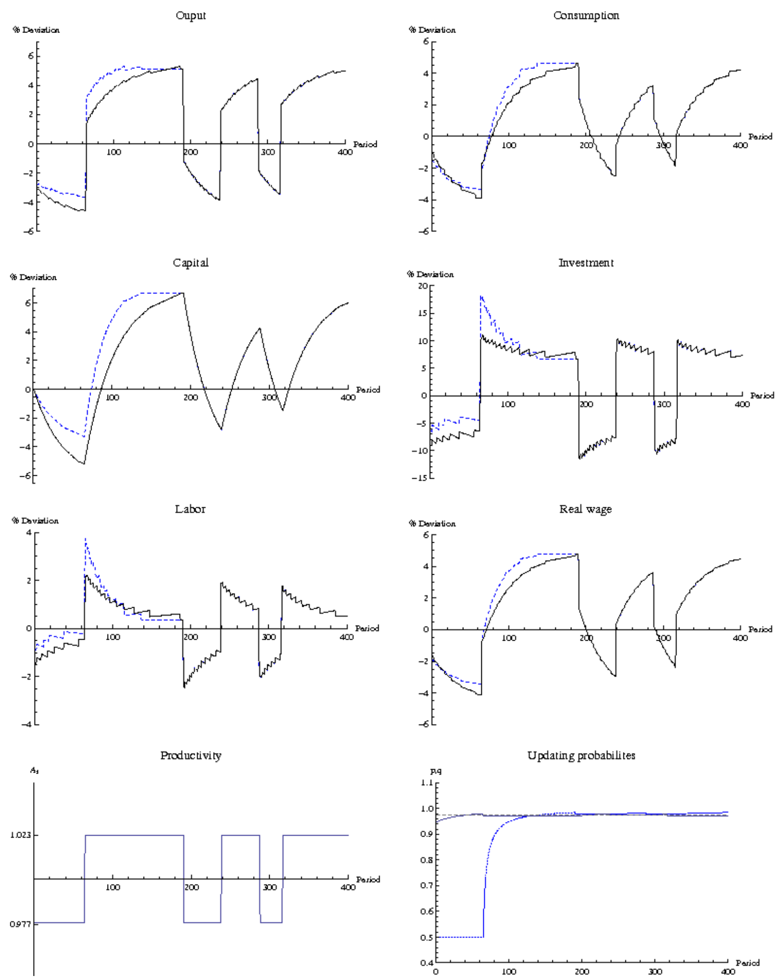

Figure 2: Individual simulation: Fast convergence. Dashed line represents deviations from steady state under learning; solid line is deviations from steady state under rational expectations.

depression state they dissave, consume and work less than under rational expectations. Similarly, once they are in expansion state which they expect to last for short period of time, Bayesian agents move more aggressively in their investment behavior that RE agents.

While in the simulation portrayed in Figure 2 the discrepancy under learning and RE vanishes within 200 periods, Figure 1 depicts a more persistent case. The realization of the random process makes updating of beliefs a long and slow process; even after 400 periods decisions made under learning are very different than under rational expectations.

Both Figure 1 and Figure 2 illustrate how the degree of dogmatism of priors affects the speed of convergence of beliefs. The bottom left panels contain the paths of p_t and q_t. Recall that prior beliefs for p in terms of counters are based on only four observations, $p_0 = \frac{2}{4}$. This prior is initially quickly updated once the productivity is in the expansion state. For example, after only 2 consecutive years (8 periods) of expansion the updated subjective probability of remaining in expansion state equals $p_t = 0.83$. As a result, the rapidly changing beliefs about transition probability matrix bring substantial revisions of optimal investment, consumption and labor supply decisions. The corresponding panels in Figure 1 and 2 illustrate these results.

I now turn to a more interesting characterization of the effects of the twisted beliefs shock.

3.2 Average difference

The average effect of pessimistically "twisted" beliefs on the economy is illustrated in Figure 3. Now, instead of looking at a particular realization of the stochastic productivity sequence, I calculate the percentage deviations of macroeconomic variables under learning from rational expectations.[8] Computed deviations are averaged across all simulations.

Figure 3 approximates impulse response functions to one-time persistent shock to beliefs. Mistakenly believing that relative to expansions duration of depression is long, agents choose to initially invest and work more, and consume less relative to what they would do if they had the correct perception of the productivity process. On average, agents would invest over 5% more, supply over 1.5% more labor, and consume 1% less under learning relative to rational expectations. These effects are substantial and long-lasting.

[8] For example, for output I compute $100 \cdot (y_t^{Learn} - y_t^{RE})/y_t^{RE}$.

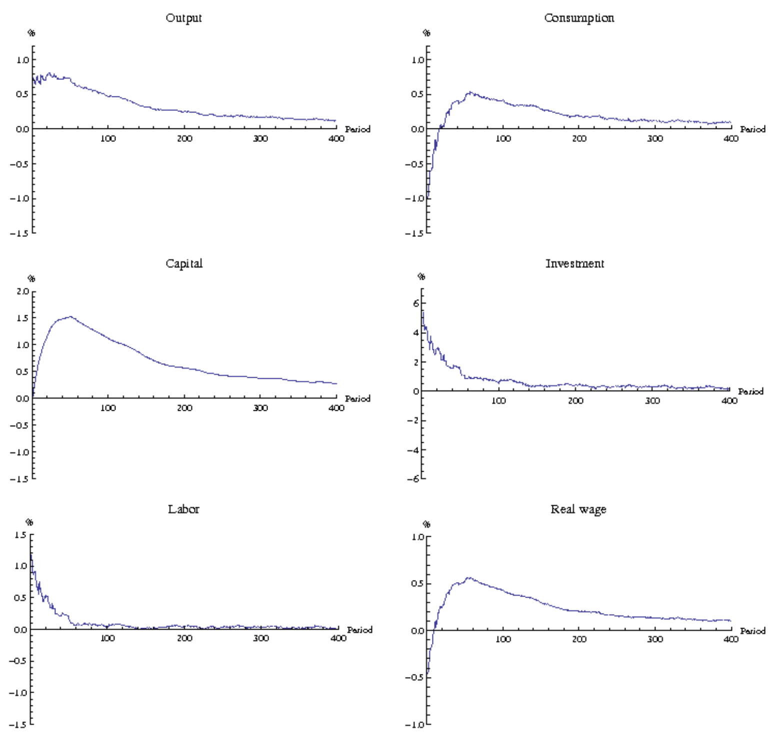

Figure 3: Average percentage difference of learning from RE. On average agents save more under learning relative to rational expectations because of pessimistically twisted beliefs.

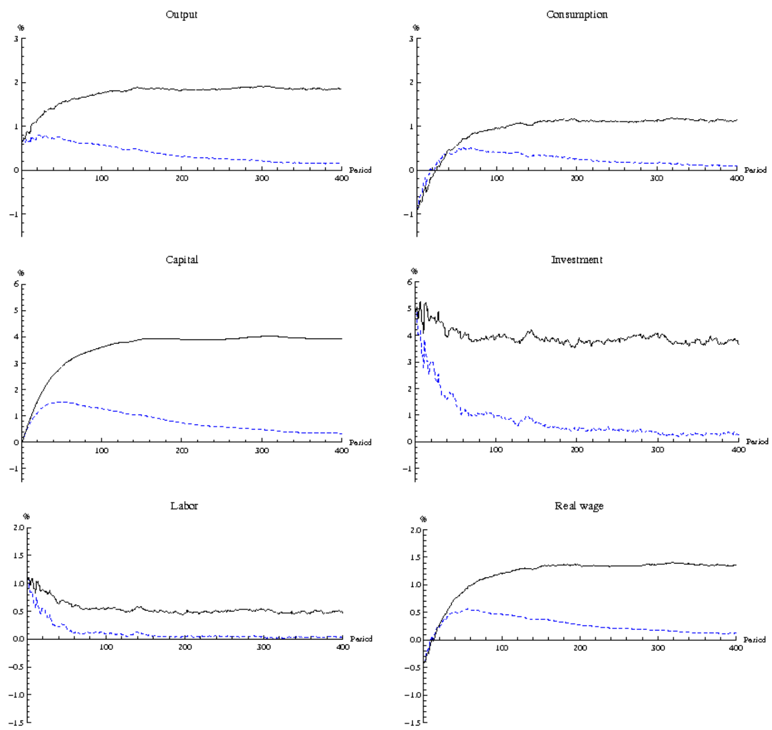

Figure 4: Learning vs no learning: average percentage difference between twisted beliefs and RE. The dashed line depicts the diffrenece between learning and RE, the solid line between no learning twisted beleifs and RE.

However, as agents update their beliefs about stochastic productivity, their perception of the distribution, Π_t, changes, getting closer to the true distribution Π. This evolution of beliefs is reflected in the evolution of decisions made by agents and evolution of macroeconomic variables. Eventually, the convergence of beliefs occurs and agents use the same decisions as under the rational expectations assumption. This transition can be seen in Figure 3. As agents update their beliefs, the average difference between learning and RE decreases. The investment and labor supply under learning are first to converge to their rational expectations values with consumption, ouput and capital following.

In Cecchetti et al. (2000) there was no learning. Without learning agents would consistently make decisions different from the rational expectations agents and the economy would settle on an alternative outcome. Figure 4 compares the evolution of macro variables for the case of agents updating and not updating their beliefs in the economy with twisted priors. The initial response to the beliefs shock is the same in both cases but while in the no learning case departures from rational expectations are permanent, in the case of learning any deviation from rational expectations has only transitory character. These differences are substantial, so learning plays an important role in understanding the effects of twisted beliefs.

Figure 3 and 4 are the illustrations of the theoretical result of the time-varying decision rule in the dynamic programming problem under adaptive learning. Recall that optimal intertemporal decision is given by equation (16)

$$u_c(c, l) = \beta \hat{E}[u_c(c', l')(r' + 1 - \delta)|s],$$

with expectations $\hat{E}$ at time t taken with respect to subjective probability distribution Π_t. As beliefs are updated, Π_t changes implying changes in intertemporal decision rule given state (s, k). This introduces time-varying paths for macroeconomic variables. For both the non learning case and the rational expectations case agents use time-invariant probability distributions, Π_0 and Π, respectively, which imply time–invariant decision rules.

Importantly, even though the one-time shock to beliefs under learning are temporary, the transition period may be long. For the baseline calibration, under learning the level of capital is at least 1% above rational expectations case for almost 30 years.[9]

[9] Modigliani (1986) remarks: "Not only was oversaving seen as having played a major role in the Great Depression, but, in addition, there was widespread fear that the problem might come back to haunt the post war era. (...) These concerns were at the base of the "stagnationist" school which was

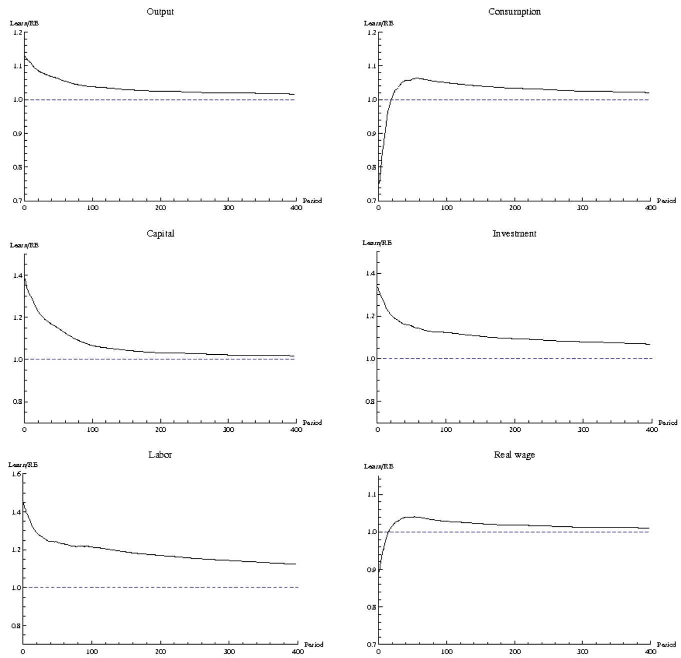

Figure 5: Volatility: Ratio of standard deviations under learning and RE.

3.3 Volatility

The time–varying decision rule under learning implies that the volatility of the macroeconomic variables might be not only the result of the stochastic fluctuations of the productivity process but also due to changes in actual decisions. To examine whether updating probabilities in case of twisted beliefs generates additional volatility in the economy, for each variable I compute the ratio of standard deviation under learning to standard deviation under rational expectations. Deviations are taken with respect to the steady state under the true data generating process. Figure 5 presents the average across all simulations ratios of standard deviations under learning and rational expectations.

The overall volatility for the case of pessimistic initial beliefs and learning is higher than for the case of rational expectations. Except for initially lower values for the consumption and for the real wage, standard deviations of macroeconomic variables under learning are persistently above standard deviations under rational expectations for the same realization of stochastic productivity. In the case of output, the average volatility under learning is initially 11% higher and remains at least 5% higher for the

prominent in the 40s and early 50s." (pp. 151) subsequent 25 years.

As agents' subjective beliefs about distribution of productivity shocks converge to the true distribution the variation in optimal policy decisions due to learning becomes smaller. As a result, the learning part of volatility economy gradually diminishes and standard deviations under learning converge to standard deviations under rational expectations driven only by stochastic productivity. Two points are worth mentioning. First, similarly to the results in the previous section, the differences in volatilities between learning and rational expectations are very persistent. Even after 50 years from the shock in the beliefs, on average, the investment standard deviation is 10% higher and the labor supply standard deviation almost 20% higher under learning comparing to rational expectations. Second, gradual convergence of beliefs implies gradual moderations of volatilities under learning.

3.4 Approximation of actual process

So far productivity shocks are drawn from the assumed distribution. The results in subsection 3.1 show that how fast learning occurs and how large the deviations are from rational expectations depends on the actual sequence of shocks. This implies that whether "twisted beliefs" and learning have important effects on macroeconomic variables depends on what productivity shocks the economy experiences. One way to answer this question is by employing a sequence of productivity states that can approximate the "true" realized process.

In this subsection I examine how the economy would behave if the sequence of productivity shocks corresponds to the one consistent with post–war U.S. experience.

To create the sequence of states I take the NBER announcements on expansions and recessions and generate accordingly the sequence of productivity states that corresponds to this historical data. Next, this sequence of states is treated as a realization of the exogenous productivity process and used in simulations of the model under rational expectations and under learning with twisted beliefs.

Figures 6 and 7 present the percentage difference and ratio of standard deviations, respectively, between learning and rational expectations in the case of NBER states. The fluctuations in the difference between learning and RE in Figure 6 are the result of two factors: the different decisions made under subjective beliefs and full information given the state of the economy represented by (k, s), and time-varying decision rules under learning. It is also the case that, for this particular realization of productivity process, Bayesian agents can update their beliefs about probability of remaining in ex-

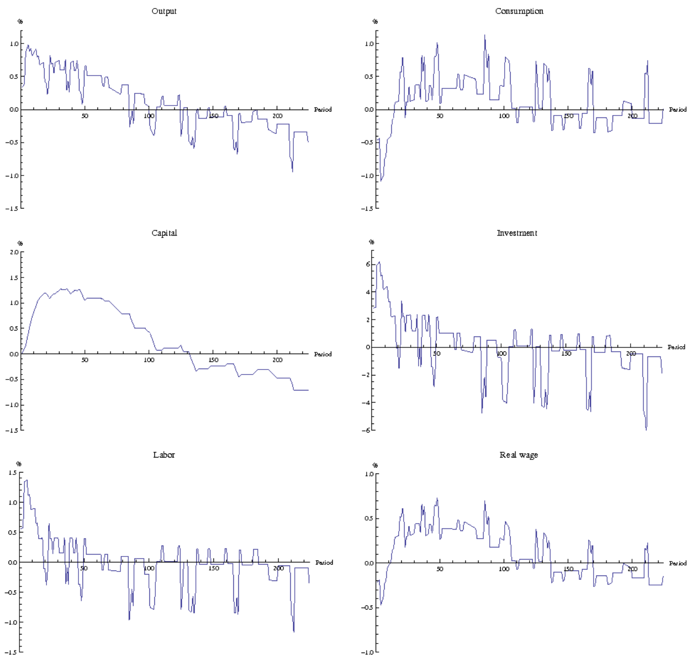

Figure 6: Percentage difference between learning and RE.

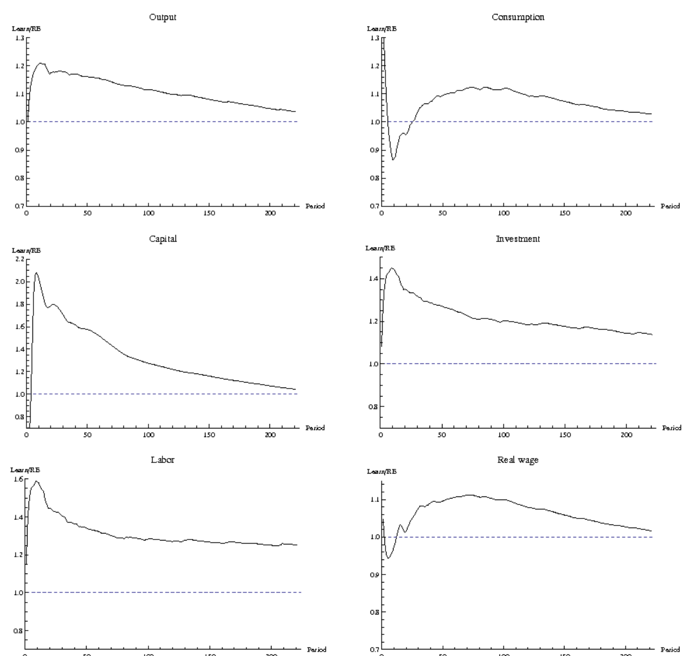

Figure 7: Volatility: Ratio of standard deviation under learning and RE.

Table 3: Prior beliefs: initial probabilities

Case	**Counters**				**Probabilities**			
	u_{11}	u_{10}	u_{01}	u_{00}	p	q	$\Pr(S_t = 1)$	$\Pr(S_t = 0)$
Baseline	2	2	2	28	0.500	0.933	0.12	0.88
Prior 1	39/5	1/5	1/5	39/5	0.975	0.975	0.50	0.50
Prior 2	4	1	1	4	0.800	0.800	0.50	0.50
Prior 3	15	1	2	18	0.880	0.950	0.29	0.71

pansion state relatively fast. As a result decisions made under learning in the periods of expansions are much closer to the one made under rational expectations. However, the probabilities concerning recession states are not updated in the direction of rational expectations probabilities. In particular, while the rational expectations agents use transition probability matrix with $p = 0.975$ and $q = 0.975$ the realized sequence of regimes corresponds to the stochastic regime switching process with probability transition matrix given by $p = 0.97$ and $q = 0.74$. This makes learning agents behave differently in recessions than RE agents.

The difference between learning and rational expectations is particularly large when considering volatility. The standard deviation of output under learning is initially 20% higher than under rational expectation and remains higher for the whole sample period. For the case of capital, the volatility is twice as high under learning as it is under rational expectations.

This result casts some light onto the real effect of a shock to beliefs on macroeconomic variables. The actual draw in the postwar data may have been one that describes a case where the effects were relatively large.

4 Robustness

4.1 Priors

The above results are derived for particular formulation of prior beliefs. In this subsection we consider the behavior of macroeconomy under alternative set of priors.

The baseline results indicate that twisted beliefs are important for levels and volatility of macroeconomic variables. To asses the robustness of these results the model is simulated with 3 alternative sets of priors that will describe alternative "twists" in agents' beliefs. Table 3 displays the prior beliefs under consideration. There are three

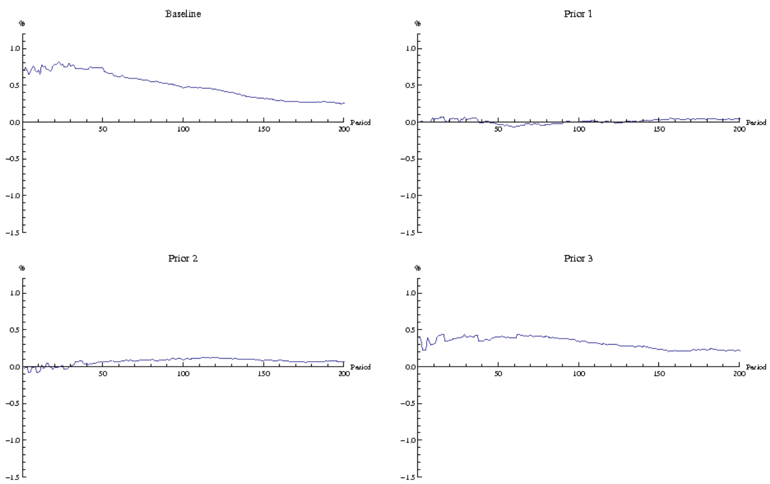

Figure 8: Priors: output average percentage difference under learning and RE

aspects of the priors for the two-state process that can affect the learning process: dogmatism (i.e. how strongly held prior beliefs are), how twisted the beliefs are with respect to persistence of the process, and with respect to unconditional probability of each state (degree of asymmetry in the process). I analyze the importance of each of these elements.

The first case considers agents having correct beliefs about business cycle properties of the stochastic process for productivity, but that do not have much trust in them. As a result, their initial beliefs will be quickly taken over by the actual realizations of the productivity shocks. On average, this should result in the same evolution of variables but introduce some volatility.

Prior 2 describes the case of mistaken beliefs about the persistence of the two states. Agents initially under-estimate the persistence of productivity: while the true probability of staying in depression/expansion is 0.975, agents believe it is 0.8. For example,when agents under-estimate productivity persistence, they try to take advantage of a lasting expansion. On the other hand, they see a depression as less of a threat as they expect it to not last as long.

In the last set of priors, agents have incorrect priors not only about persistence of states but also about their relative frequencies. As in case 2, agents under-estimate the probability of remaining in the current state but now agents see recessions as relatively more persistent than expansions: the recession state is expected to last almost 5 years while expansion to last just over 2 years. This set of priors is consistent with NBER

Table 4: Maximum percentage difference of deviations from steady state under learning and RE for different set of priors.

Variable		**Baseline**	**Prior 1**	**Prior 2**	**Prior 3**
Output	%	0.81	0.08	0.13	0.44
	quarter	23	252	112	16
Consumption	%	0.54	0.09	0.12	0.30
	quarter	57	38	4	101
Capital	%	1.52	0.14	0.24	0.86
	quarter	51	285	124	81
Investment	%	5.37	0.62	0.61	3.09
	quarter	2	16	37	1
Labor	%	1.17	0.09	0.12	0.68
	quarter	2	16	33	1
Real wage	%	0.57	0.06	0.09	0.31
	quarter	57	291	135	101

The numbers reported below percentage difference are the periods in which the maximum deviation occured.

dates for recessions and expansions for the period of 1929:2 – 1938:2.

Each of the cases considers an alternative "twist" in beliefs. Figure 8 portrays the average percentage difference for output under learning and rational expectations. The results suggest that, on average, the degree of dogmatism in the case of correctly specified priors does not introduce quantitatively important differences between decisions under learning compared to the ones taken under rational expectations. A similar result holds if we vary the degree perceived persistence of the stochastic process. However, if agents start with mistaken beliefs concerning relative unconditional probability of expansions and depressions, i.e. they see states being asymmetrically distributed; the choice of consumption, investment and labor supply will be different than if agents had the correct distribution.

Table 4 presents the average maximum percentage difference in macro variables under alternative informational assumptions. Only the last set of priors has quantitatively important effect on the behavior of the economy.

Table 5: Maximum likelihood estimates of the productivity process

	p	q	A_H	A_L
Estimate	0.970	0.657	0.234	−0.782
Standard error	0.071	0.256	0.139	0.685

4.2 Asymmetric shock process

In this section I consider an alternative specification for the data generating process. The purpose of the benchmark calibration was to use standard real business cycle model parameters and examine how a departure from the assumption of rational expectations agents and the introduction of Bayesian agents with twisted beliefs affects the macroeconomy. However, the often-made assumption in the RBC literature of symmetric distribution of expansions and recessions may not be the correct representation of the true stochastic productivity process. It is shown in Section 3.4 that the post-war U.S. data on recessions are better described by asymmetric process with relatively longer expansions and shorter recessions. In this section, I use this representation of the stochastic process as the true data generating mechanism for productivity.

To derive formally the cyclical properties of U.S. productivity I consider a hidden Markov model for productivity and apply Hamilton's (1989) Markov switching model to quarterly post-war US productivity data for the period 1948:1-1982:4.[10] The estimated process for $\Delta \log A_t$ is in Table 5.

The estimated model suggests asymmetry in both relative persistence and relative severity of expansions and depressions. The expansion state is much more persistent than the recession state which corresponds to U.S. post-war experience. The recession state is much more severe in terms of productivity decline. Note that these estimates are qualitatively similar to the ones for consumption (used by Cecchetti, et al. (2000) and Cogley and Sargent (2008b)) and U.S. GNP (estimated by Hamilton (1989) and Kim and Nelson (1999)).

To calibrate the stochastic process for productivity used in this section, I use the estimated probabilities p, q as elements of the true, rational expectations transition probability matrix Π. In order to keep the standard deviation of the productivity series at 0.007, the productivity levels in both regimes are set to $z_H = 0.01$ and $z_L = -0.023$. Under the alternative specification of productivity process, the business cycle properties

[10] The data for productivity is quality adjusted Solow residual computed and kindly provided by Raul Santaeulalia-Llopis.

Table 6: Standard deviations—asymmetric and symmetric case.

Variable	RE symmetric case	RE asymmetric case
Output	1.231	1.193
Consumption	0.462	0.248
Hours	0.593	0.723
Investment	3.655	4.342
Capital	0.250	0.185

Note: Average (400 simulations) percentage deviations from Hodrick-Prescott trend.

of this model are different than in Prescott (1986) as Table 6 reports. In particular, the standard deviations of consumption and capital are significantly below ones generated by the symmetric stochastic process and observed in the U.S. data. However, according to the estimation result, the asymmetric process serves as a better description of the data, and matches better the duration of regimes. Under the above parameterization of asymmetric stochastic process, the expected duration of expansions and recessions is 33 and 3 quarters, respectively. The actual post–war U.S. recessions lasted, on average, 10 months, while expansions were almost 14 quarters long.[11]

Figure 9 presents the evolution of aggregate variables under the asymmetric stochastic process for an individual simulation. The prior beliefs for Bayesian agents are set to the baseline pessimistically "twisted" case. Agents quickly learn the probability of remaining in the expansion state, p, but cannot update beliefs about persistence of recession state. As the economy visits the recession state infrequently, agents cannot update their initial beliefs and, hence, persistently over-estimate q. This slow convergence of probability of the “bad” state resembles the result in Cogley and Sargent (2008b).

Figures 10 and 11 portray the average results for relative performance under learning and rational expectations in case of deviations from steady state and volatility.

Similarly to the symmetric productivity process, the percentage differences between learning and rational expectations are significant and long lasting. The pessimistic twist in beliefs induces over—accumulation of capital, higher labor supply and an initial decrease in consumption. The updating of beliefs brings investment and labor supply decisions under learning to their rational expectations counterparts but the average responses of output, capital, or consumption to a belief shock deviate from rational expectations longer, as can be seen in Figure 10.

[11] Given the standard errors one cannot statistically reject the hypothesis of median duration of expansion equal to observed data.

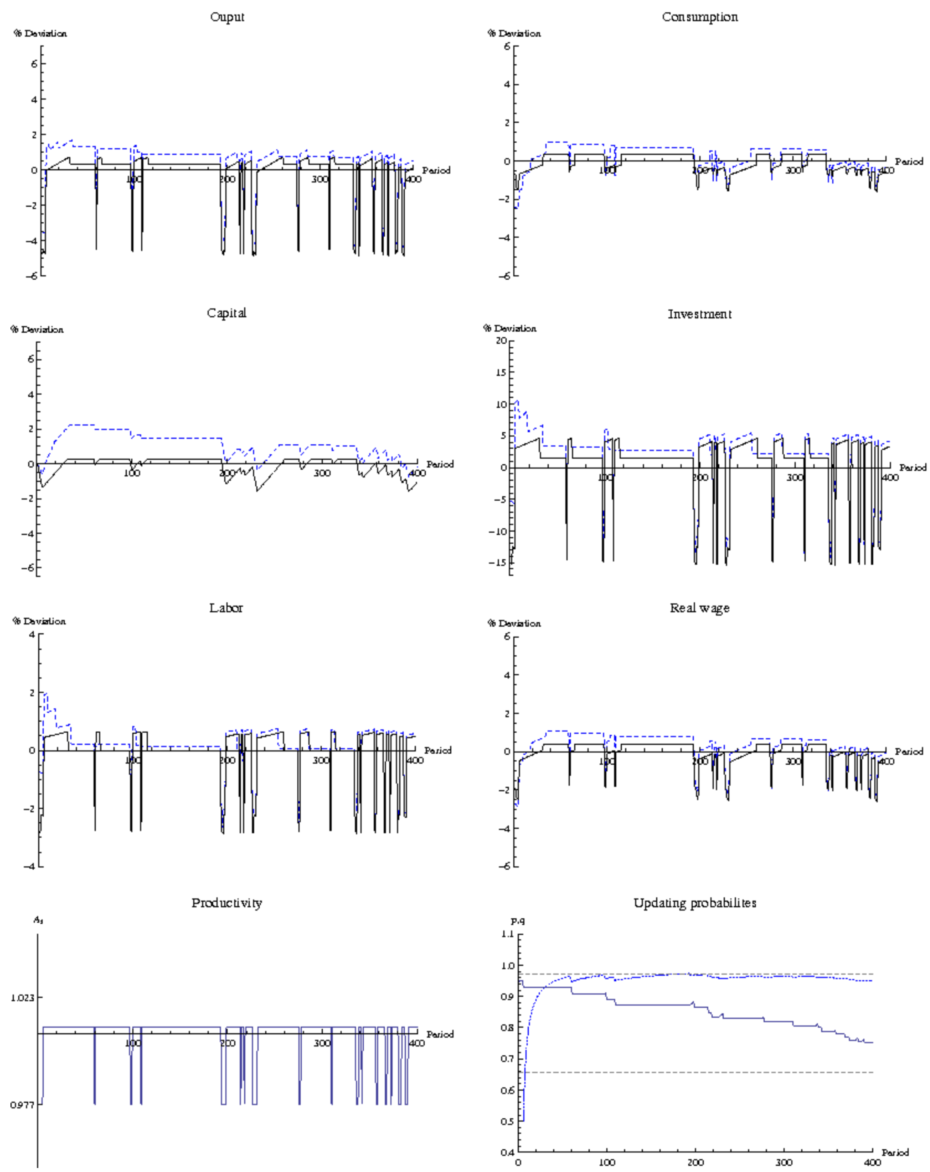

Figure 9: Asymmetric process - individual simulation. Dashed line represents deviations from steady state under learning; solid line is deviations from steady state under rational expectations.

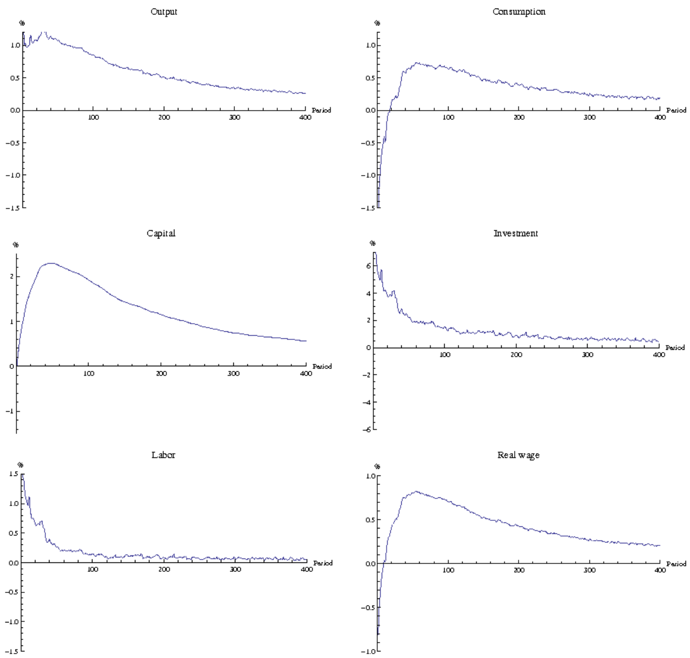

Figure 10: Asymmetric process - average percentage diffrence between learning and RE.

The comparison of results presented in Figure 11 and Figure 5 indicates that the asymmetry of the productivity distribution does affect the volatility of macroeconomic variables under learning relative to volatilities under rational expectations. While the pattern of initially higher standard deviations and slow convergence is still present, the quantitative difference between learning and rational expectations is now more pronounced. Following a beliefs shock, the average standard deviation of output in the economy with Bayesian agents is over 15% higher than in the economy populated by rational expectations agents. The difference is even larger for consumption, capital, or investment and, in all cases, is persistent. Most importantly, however, learning introduces a significant degree of moderation in volatilities. Within 50 years from the twist in the beliefs, the relative volatility of consumption decreases by 40% and of labor supply by 25%.

These results suggest that beliefs shock and learning could be part of an explanation of the Great Moderation.

Cecchetti et al. (2000) and Cogley and Sargent (2008b) use different conceptions of the shock process from the standard approach as described by Auroba et al. (2006). This means that business cycle properties will also be different, but I wanted to show

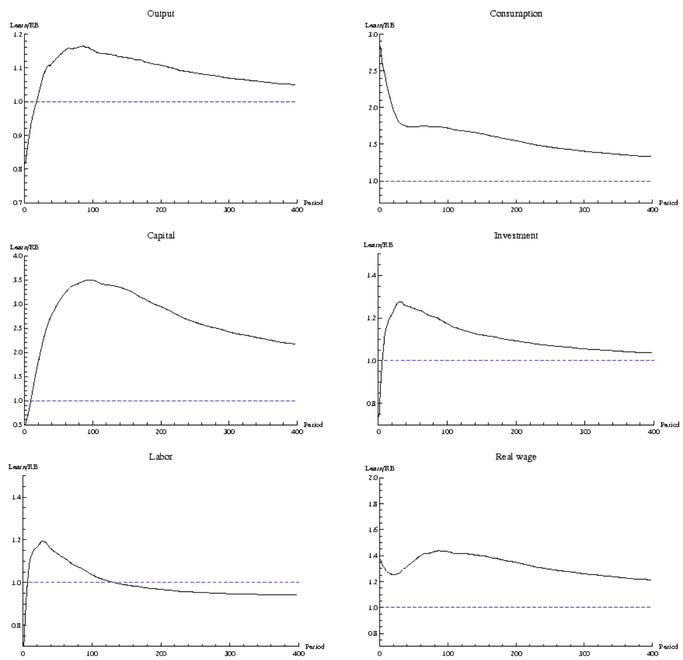

Figure 11: Asymmetric process - ratio of standard deviations under learning and RE.

how the results would differ under a process closer to the one used in models of asset pricing and learning.

5 Conclusions

I studied the effects of "shattered beliefs" in a dynamic stochastic general equilibrium model with Bayesian learning. The main point is that a beliefs-twisting event is likely to alter agents' behavior even if the underlying processes governing the economy remain unchanged. This is because the perceived distribution of the driving stochastic process differs from the true data generating mechanism.

The learning guarantees that any effects will be temporary, yet the effects of a beliefs-twisting event like the Great Depression are found to be substantial and long-lasting. Even after 50 years, the decisions made under subjective expectations may be markedly different from the ones taken under rational expectations. This is because the observation of macroeconomic data corrects the twisted beliefs very slowly. This mirrors the findings of Cogley and Sargent (2008b) in their partial equilibrium asset pricing framework.

If a beliefs twisting event can have large effects on the economy, it may be of interest to study other such events. In particular, one might expect larger and more frequent beliefs-twisting events in developing countries.

This framework can be also used to analyze the behavior of an economy and agents' beliefs in case of a process-twisting event. Any changes in the stochastic processes driving the economy are often not directly observable causing subjective and "correct" expectations to differ. As this paper shows, for sufficiently large differences it may take a long time for agents to learn the new process governing the economy.

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7 Appendix

7.1 AR(1) representation of 2-state Markov regime switching model

Calibrating parameters of two–state Markov process to obtain desired characteristics of the process.

AR(1) representation of 2–state Markov regime switching process S_t with $p = Prob(S_{t+1} = 1|S_t = 1)$ and $q = Prob(S_{t+1} = 0|S_t = 0)$ is

$$S_t = \lambda_0 + \lambda_1 S_{t-1} + v_t,$$

with

$$\begin{aligned} \lambda_0 &= 1 - q, \\ \lambda_1 &= p + q - 1, \end{aligned}$$

and error term, ν_t, with zero mean and variance σ_ν^2.

Therefore, the AR(1) representation of productivity process $z_t = z_H S_t + z_L(1 - S_t)$ is

$$z_t = (z_H - z_L)\lambda_0 + z_L(1 - \lambda_1) + \lambda_1 S_{t-1} + (z_H - z_L)\nu_t,$$

where the error term ν_t, has mean zero and variance

$$\sigma_\nu^2 = p(1-p)\frac{\lambda_0}{1-\lambda_1} + q(1-q)\left(1 - \frac{\lambda_0}{1-\lambda_1}\right).$$

Parameter values for p, q, z_H, and z_L are set to match the properties of the productivity process.