Macroeconometrics: Final Exam

Paris School of Economics - Winter 2010

Jacek Suda

February 23, 2010

Solve all problems. The due date is the 23:59 on 24 February 2010. Email me if something is not clear. All answers must be type in. Good luck.

Question 1

1. Choose two models (out of three below) and give their state-space representation:

   (a) AR(2) model

   $$\begin{aligned} y_t &= \alpha + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \eta_t, \\ &\quad \eta_t \sim iid\, N(0, \sigma^2). \end{aligned}$$

   Compute also initial state vector $\beta_0 = E[\beta_t]$.

   (b) Time-varying parameter model

   $$\begin{aligned} y_t &= x_{1t}\beta_{1t} + x_{2t}\beta_{2t} + u_t, \quad u_t \sim iid\, N(0, \sigma^2), \\ \beta_{i,t} &= \beta_{i,t-1} + v_{it}, \quad v_t \sim iid\, N(0, \sigma_i^2). \end{aligned}$$

   (c) Unobserved component model

   $$\begin{aligned} y_t &= \tau_t + c_t, \\ \tau_t &= \delta + \tau_{t-1} + v_t, \quad v_t \sim iid\, N(0, \sigma_v^2), \\ c_t &= \phi_1 c_{t-1} + \phi_2 c_{t-2} + u_t, \quad u_t \sim N(0, \sigma_u^2), \end{aligned}$$

   where the roots of $\phi(z) = 1 - \phi_1 z - \phi_2 z^2 = 0$ lie outside the complex unit circle.

2. Briefly discuss how you would estimate a model put in state space form using maximum likelihood. That is, describe how would compute the log-likelihood function and how you would maximize the log-likelihood.

Question 2

Consider the following dynamic simultaneous equations model

$$\begin{aligned} y_{1t} &= \beta_{12} y_{2t} + \gamma_{11} y_{1t-1} + \gamma_{12} y_{2t-1} + \varepsilon_{1t} \\ y_{2t} &= \beta_{21} y_{1t} + \gamma_{21} y_{1t-1} + \gamma_{22} y_{2t-1} + \varepsilon_{2t} \end{aligned}$$

1. Write down this model as a structural VAR model, *i.e.* in a matrix form of

$$\mathbf{B}\mathbf{y_t} = \mathbf{\Gamma}\mathbf{y_{t-1}} + \varepsilon_\mathbf{t},$$

   where $\mathbf{y_t} = (y_{1t}, y_{2t})'$, $\varepsilon_\mathbf{t} = (\varepsilon_{1t}, \varepsilon_{2t})'$, and $\mathbf{B}$ and $\mathbf{\Gamma}$ are matrices of parameters. It is assumed that error term satisfies $\varepsilon_\mathbf{t} \sim iid(0, \mathbf{\Sigma})$, and $\mathbf{\Sigma}$ is diagonal with σ_i^2 $(i = 1, 2)$.

2. Determine the reduced form of the VAR model

$$\mathbf{y_t} = \mathbf{A}\mathbf{y_{t-1}} + \mathbf{e_t}$$

   and use it to show that the parameters of the structural VAR(1) are not identified without further restrictions.

3. What kind of restrictions are usually put on the parameters of the structural VAR(1) in order to achieve identification?

4. Suppose $\mathbf{B}^{-1}$ is lower triangular matrix. What does this imply for the structural model? How can this be used to identify the VAR model?

5. Suppose you want check if y_{1t} Granger-causes y_{2t}. How can you do it using the reduced-form VAR(1)? If you find that y_{1t} Granager causes y_{2t}, what does this imply for the parameters of our structural VAR model?

Question 3

Consider the following linearized version of a simple DSGE model:

$$\begin{aligned} x_t &= \rho x_{t-1} + u_{x,t} \\ y_t &= E_t(y_{t+1}) - \frac{1}{\gamma}[r_t - E_t(\pi_{t+1})] + u_{y,t} \\ \pi_t &= E_t(\pi_{t+1}) + \kappa[y_t - x_t] + u_{\pi,t} \\ r_t &= \phi_\pi \pi_t \end{aligned}$$

The goal is to put this system into a space-space form and estimate. Do the following steps:

1. Substitute in the interest rate into the Euler equation.

2. Solve the model using the method of undetermined coefficients:

   (a) Guess that the model can be put in the form

$$\begin{aligned} x_t &= \rho x_{t-1} + u_{x,t} \\ y_t &= a x_t + u_{y,t} \\ \pi_t &= b x_t + u_{\pi,t} \end{aligned}$$

(b) Substitute in the guessed form of solution (ignoring shocks $u_{y,t}$ and $u_{\pi,t}$ for now) into structural equations. Use $E_t(x_{t+1}) = \rho x_t$. You should have a system in x_t with a and b as unknown coefficients.

(c) Solve for a and b using method of undetermined coefficients
Hint: If, for example, the guessed solution is of the form $\pi_t = bx_t$ (again ignore shocks) and after substituting it in into structural equation you get

$$bx_t = b\rho x_t + \kappa[ax_t - x_t],$$

then method undetermined coefficients requires equating coefficients on the right and left hand side

$$b = b\rho + \kappa[a - 1]$$

to find the solution.

(d) Write down the solution of the model with shocks, *i.e.* add $u_{y,t}$ to the solution of $y_t = ax_t$ and u_{π_t} to $\pi_t = bx_t$.

3. Write down the system of equations in the state-space form

$$\begin{aligned} X_t &= AX_{t-1} + Cu_t \\ Z_t &= DX_t + v_t \end{aligned}$$

Note that x_t is an unobservable productivity.

4. Write briefly how would you estimate coefficients of this simple DSGE model.