Macroeconometrics: Final Exam

Paris School of Economics - Fall 2011

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Solve all problems. The due date is the 23:59 on 5 December 2011. Email me if something is not clear. All answers must be type in. Good luck.

Question 1

Consider an ARMA(1,1) model:

$$y_t = \phi y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}, \qquad \varepsilon_t \sim iidN(0, \sigma^2).$$

1. What are the stationarity conditions for this process?
2. What are the invertibility conditions for this process?
3. Express the process in the Wold form and specify Wold form coefficients in terms of ϕ and θ.
4. Compute the variance and calculate the autocorrelation function, ACF.
5. Consider the case in which $\phi = -\theta$.
   (a) How does the ACF look like?
   (b) What can you say about this process?

Question 2

1. Give the state-space representation of the following models:
   (a) AR(2) model

$$\begin{aligned} y_t &= \alpha + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \eta_t, \\ &\eta_t \sim iid\, N(0, \sigma^2). \end{aligned}$$

   Compute also the initial state vector $\beta_0 = E[\beta_t]$.

   (b) Time-varying parameter model

$$\begin{aligned} y_t &= x_{1t}\beta_{1t} + x_{2t}\beta_{2t} + u_t, \quad u_t \sim iid\, N(0, \sigma^2), \\ \beta_{i,t} &= \beta_{i,t-1} + v_{it}, \quad v_t \sim iid\, N(0, \sigma_i^2). \end{aligned}$$

2. Briefly discuss how you would estimate a model in a state space form using maximum likelihood. That is, describe how you would compute and maximize the log-likelihood function.

Question 3

Consider the unobserved component model with AR(2,1) structure for the transitory component, c_t:

$$
\begin{aligned}
y_t &= \tau_t + c_t, \\
\tau_t &= \mu + \tau_{t-1} + \eta_t, \qquad \eta_t \sim N(0, \sigma_\eta^2) \\
c_t &= \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon_t + \theta \varepsilon_{t-1}, \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \\
& \quad cov(\eta_t, \varepsilon_t) = \sigma_{\eta\varepsilon}
\end{aligned}
$$

1. Under what conditions $c_t \sim I(0)$?

2. Show that without making any assumption about the correlation between permanent and transitory component, $\sigma_{\eta\varepsilon}$, this model is not identified.

3. Show that under the assumption $\sigma_{\eta\varepsilon} = 0$, the model is identified.

4. What does this assumption, i.e. $\sigma_{\eta\varepsilon} = 0$, mean?

5. Cast the model in a state space form and write how it can be estimated.