Lecture: New-Keynesian Model

Makroekonomia / Macroeconomics

SD SGH

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Flexible vs. sticky prices

- Central assumption in the new classical economics:
  - Prices (of goods and factor services) are fully flexible
  - Classical dichotomy: money is superneutral and monetary policy has no real effects
  - Consequences for models: while analysing business cycle behaviour we can abstract from money and nominal variables
- (New) Keynesian economics:
  - Prices are sticky, i.e. they adjust sluggishly to macroeconomic shocks (including monetary shocks)
  - Classical dichotomy does not hold: monetary policy has real effects
  - Also, additional propagation channels for other shocks
  - Consequences for models: money and nominal variables important

Sticky prices: empirical evidence

- Testing for price stickiness:
  - Frequency and size of price changes (scanner data)
  - Panel data
  - Some issues: sales, promotions, consumption goods,...
- Results for price duration:
  - US: average time between price changes is 2-4 quarters (Blinder et al., 1998; Bils and Klenow, 2004; Klenow and Kryvstov, 2005)
  - Euro area: average time between price changes is 4-5 quarters (Rumler and Vilmunen, 2005; Altissimo et al., 2006)
  - Poland: average time between price changes is 4 quarters (Macias and Makarski, 2013)
- The higher inflation, the more frequently price changes occur (Macias and Makarski, 2022)
- Cross-industry heterogeneity
  - Prices of tradables less sticky than those of nontradables
  - Retail prices usually more sticky than producer prices

Why are prices sticky?

- Lucas (1972): imperfect information
  - A firm observing an increase in the price of its product does not know whether it reflects a change in the product's relative price (possibly due to higher demand) or a change in the aggregate price level
  - In the first case a firm should raise its output, while in the latter case it should not
  - The rational response under uncertainty: increase output somewhat
  - Extensions: rational inattention (Sims, 2003; Maćkowiak and Wiederholt, 2009)
- Costs of changing prices (explicit or implicit):
  - Menu costs
  - Explicit contracts which are costly to renegotiate
  - Long-term relationships with customers
- 'Good' causes of price stickiness: in a stable economic environment agents trust in price stability

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The simplest New Keynesian model - basic features

- In a nutshell - the RBC model
  - Stochatic dynamic general equilibrium model
- with
  - two stages of production, at one of them firms are monopolistically competitive and set their prices
    - but some prices sometimes are sticky,
  - a monetary authority/central bank operating via an interest rate feedback rule,
  - and many simplifications:
    - no trend productivity growth, only stationary stochastic shocks, no capital;
    - (but we can and do add them)
- Key result: monetary policy has real effects

Households I

- Rent labour l_t for the real wage w_t.
- Own firms and so get their profits Div_t.
- Hold nominal bonds B_t paying a nominal and risk-free (i.e. determined in the previous period) interest rate R_{t-1} (in gross terms)
- Make optimal consumption-savings (by adjusting bond holdings) and work-leisure decisions, given the price in period t, P_t.
- Pay lump-sum taxes T_t.

Households II

- Households maximise their expected lifetime utility ($\psi_t -$ preference shock)

$$U_t = E_t \sum_{j=0}^{\infty} \beta^{t+j} \psi_{t+j} \left(\frac{c_{t+j}^{1-\sigma}}{1-\sigma} - \vartheta \frac{l_{t+j}^{1+\varphi}}{1+\varphi} \right) \quad (1)$$

- Budget constraint (Lagrange multiplier λ_t)

(2)

- No-Ponzi game condition

(3)

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$$c_{t+j} + \frac{B_{t+j}}{P_{t+j}} = w_{t+j} l_{t+j} \quad (2)$$

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- No-Ponzi game condition

$$B_{t+j} \geq \tilde{B} \text{ (small enough)} \tag{3}$$

Households' optimisation

- Lagrange function:

$$\mathcal{L}_t =$$

- First order conditions:

$$c_t : \tag{4}$$

$$l_t : \tag{5}$$

$$B_t : \tag{6}$$

and the transversality conditions.

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- First order conditions:

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$$B_t : \quad \frac{\lambda_t}{P_t} = E_t \left[R_t \frac{\lambda_{t+1}}{P_{t+1}} \right] \tag{6}$$

and the transversality conditions.

Households: Equilibrium conditions

- Consumption labour choice: (4)+(5)

- Intertemporal condition (Euler equation): (4)+(6)

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- Consumption labour choice: (4)+(5)

$$\vartheta l_t^{\varphi} = c_t^{-\sigma} w_t$$

- Intertemporal condition (Euler equation): (4)+(6)

$$\frac{1}{R_t} = \beta E_t \left[\frac{\psi_{t+1} c_{t+1}^{-\sigma}}{\psi_t c_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right]$$

Log-linear approximation

- Due to nonlinearities and presence of expectations, the model does not have a closed-form solution
- Standard technique: log-linear approximation of the model equations around the (non-stochastic) steady-state.
- We use the following log-linearisation technique

$$x_t^a$$

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where $\hat{x}_t = \log \frac{x_t}{x}$,

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$\Rightarrow$ using 1$^{\text{st}}$-order Taylor expansion of $e^{a\hat{x}_t}$ around $\hat{x}_t = 0$,

$$e^{a\hat{x}_t} \approx e^{a \cdot 0} + \left. \frac{\partial e^{a\hat{x}_t}}{\partial \hat{x}_t} \right|_{\hat{x}_t = 0} (\hat{x}_t - 0)$$

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$$= x^a e^{a\hat{x}_t} \approx x^a \left(1 + a\hat{x}_t\right)$$

where $\hat{x}_t = \log \frac{x_t}{x}$,
$\Rightarrow$ using 1$^{\text{st}}$-order Taylor expansion of $e^{a\hat{x}_t}$ around $\hat{x}_t = 0$,

$$e^{a\hat{x}_t} \approx e^{a\cdot 0} + \left.\frac{\partial e^{a\hat{x}_t}}{\partial \hat{x}_t}\right|_{\hat{x}_t=0} (\hat{x}_t - 0) = 1 + ae^{a\cdot 0} \cdot \hat{x}_t = 1 + a\hat{x}_t$$

Households: Log-linearised equations

- Consumption labour choice

$$\varphi \hat{l}_t + \sigma \hat{c}_t = \hat{w}_t \tag{7}$$

- Intertemporal (bonds)

$$R_t - E_t \hat{\pi}_{t+1} = \sigma \left(E_t \hat{c}_{t+1} - \hat{c}_t \right) + \left(1 - \rho_\psi \right) \hat{\psi}_t \tag{8}$$

Firms

- Two stages of production:
  - Final-goods firms produce output by combining intermediate goods
  - Intermediate-goods firms produce using labour

- Contrary to the RBC model, final-goods production is non-trivial since intermediate goods are not perfect substitutes. Therefore, the final output is not a simple sum of intermediate goods production.

Final-goods firms I

- Final-goods firms produce according to the CES production function (Dixit-Stiglitz aggregator):

$$y_t = \left(\int_0^1 y_t(i)^{\frac{1}{1+\mu}} di \right)^{1+\mu} \tag{9}$$

where:

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where:

- The continuum of intermediate-goods firms (indexed by i) is normalised to 1
- $y_t(i)$ is output produced by intermediate-goods firm i
- $\mu > 1$ is a mark up over marginal cost and $\frac{1+\mu}{\mu}$ is the elasticity of substitution between individual intermediate goods

- Note: When $\mu \to 0$, y_t is a simple sum of intermediate products (like in the RBC model, where all producers are perfectly competitive)

Final-goods firms II

- Maximisation problem of final-goods firms:

$$\max_{y_t,(y_t(i))_{i\in[0,1]}} \left\{ P_t y_t - \int_0^1 P_t(i) y_t(i) di \right\} \tag{10}$$

subject to production function constraint (9)

- Final-goods firms are perfectly competitive, so
  - they maximise their profits by choosing the inputs $\{y_t(i)\}_{i\in[0,1]}$ and output y_t, taking all prices ($P_t(i)$ and P_t) as given,
  - in equilibrium profits are zero.

Final-goods firms III

- Substituting (9) into (10) the maximisation problem becomes

$$\max_{(y_t(i))_{i\in[0,1]}} \left\{ P_t \left(\int_0^1 y_t(i)^{\frac{1}{1+\mu}} di \right)^{1+\mu} - \int_0^1 P_t(i) y_t(i) di \right\}. \tag{11}$$

- The first order condition can be written as:

$$y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{\frac{-(1+\mu)}{\mu}} y_t. \tag{12}$$

- Equation (12) defines the demand for intermediate input i.

Intermediate-goods firms

- Firm i production function:

$$y_t\left(i\right) = z_t l_t\left(i\right). \tag{13}$$

- Productivity z_t is common to all firms and $\ln z_t$ follows the first-order autoregressive process:

$$\ln z_t - \ln \bar{z} = \rho\left(\ln z_{t-1} - \ln \bar{z}\right) + \varepsilon_{z,t} \quad , \tag{14}$$

where $0 \leq \rho < 1$ and $\varepsilon_{z,t} \sim iid(0, \sigma^2)$.

- Labour input is rented from households, technology is available for free.
- Prices are set according to the Calvo (1983) mechanism.
- For clarity before we move on to the profit maximisation problem we first find the cost function $c(y(i))$ (solve the cost minimisation problem).

Cost function I

- Since total cost (only labour input) equals $w_t l_t(i)$, using the production function $y_t(i) = z_t l_t(i)$ we get the following formula for the cost function
$$c(y_t(i)) = \frac{y_t(i)}{z_t} w_t.$$

- The marginal cost is
$$mc_t\left(y_t\left(i\right)\right) = \frac{w_t}{z_t}$$
Notice that the marginal cost is a constant function of $y_t(i)$. We can use it in the profit maximisation problem.

Cost minimisation: Log-linearised equilibrium conditions.

- Marginal cost

Cost minimisation: Log-linearised equilibrium conditions.

- Marginal cost

$$\hat{mc}_t = \hat{w}_t - \hat{z}_t \tag{15}$$

Price setting with flexible prices I

- Maximisation problem of intermediate-goods firm i is

$$\max_{P_t(i),y_t(i)} \left(P_t(i) - mc_t\right) y_t(i)$$

subject to the demand function (12).

- Using the demand function it can be written as

$$\max_{P_t(i)} \left\{ \left(P_t(i) - mc_t\right) \left(\frac{P_t(i)}{P_t}\right)^{\frac{-(1+\mu)}{\mu}} y_t \right\}.$$

- Each intermediate-good firm takes the economy-wide price level P_t and output y_t as given.

Price setting with flexible prices II

- Rearranging the maximisation problem

$$\max_{P_t(i)} \left\{ \left(P_t(i)^{-\frac{1}{\mu}} - mc_t P_t(i)^{-\frac{1}{\mu}-1} \right) \left(\frac{1}{P_t} \right)^{\frac{-(1+\mu)}{\mu}} y_t \right\}$$

- First-order condition yields:

$$\left(-\frac{1}{\mu} P_t(i)^{-\frac{1}{\mu}-1} + \frac{(1+\mu)}{\mu} mc_t P_t(i)^{-\frac{1}{\mu}-2} \right) \left(\frac{1}{P_t} \right)^{\frac{-(1+\mu)}{\mu}} y_t = 0$$

- and after simplification

$$P_t(i) = (1+\mu)\, mc_t \tag{16}$$

- Imperfectly competitive intermediate-good firms set their prices as a (constant) mark-up, equal to $1+\mu$, over marginal costs.
- Note that since neither mark-ups nor marginal costs are firm-specific, all intermediate-goods firms choose the same prices.

Price setting with sticky prices I

- Calvo scheme: Each period each firm with probability $1-\theta$, $\theta \in [0,1]$ receives a signal to reoptimise its prices (i.e., a constant proportion $1-\theta$ of firms reoptimises) and chooses $P_t^{new}(i)$.
- Otherwise, there are three options in the literature:
  - it keeps its price unchanged (problem with the steady state),
  - indexes to the steady state inflation, i.e. $P_t(i) = P_{t-1}(i)\bar{\pi}$ (no hump shape in monetary policy irf),
  - indexes according to the following formula $P_t(i) = P_{t-1}(i)\pi_t^{\zeta}$, where $\pi_t^{\zeta} = (1-\zeta)\bar{\pi} + \zeta\pi_{t-1}$ (allows for hump shaped monetary policy irf, nests the first two).
- Firms allowed to reset their price take into account that they may not be allowed to do so in the future.
- The probability that in period $t+j$ the price of intermediate-goods firm i is not reoptimised equals θ^j
- The expected time of a given price remaining not reoptimised is $(1-\theta)^{-1}$.

Price setting with sticky prices II

- Intermediate-good firm i chooses $P_t^{new}(i)$, $\{y_{t+j}(i)\}_{j=0}^{\infty}$ to maximise:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \Lambda_{t,t+j} \left(\frac{P_t^{new}(i)\pi_{t,t+j}^{\zeta}}{P_{t+j}} - mc_{t+j} \right) y_{t+j}(i)$$

  subject to the demand function (12).

- Note:
  - Profit maximisation is dynamic: firms take into account that they may not have a chance to reset their prices in the future
  - Firms are owned by households, so they discount their future profits by the discount factor $\beta^j \Lambda_{t,t+j}$, where $\Lambda_{t,t+j} = \psi_{t+j} c_{t+j}^{-\sigma} / \psi_t c_t^{-\sigma}$.

Price setting with sticky prices III

- First-order condition:

$$E_t \sum_j (\beta\theta)^j \Lambda_{t,t+j} \left(\frac{P_t^{new}(i)}{P_{t+j}} - (1+\mu)\, mc_{t+j} \right) y_{t+j}(i) = 0 \qquad (17)$$

- First-order condition (17) is the same for each firm allowed to reset its price
- Therefore, all firms allowed to reoptimise at time t choose the same price, which we denote by P_t^{new}

Price setting with sticky prices IV

- First-order condition (17) is the same for each firm allowed to reset its price
- Therefore, all firms allowed to reoptimise at time t choose the same price, which we denote by P_t^{new}
- The aggregate price level P_t is then:

$$\begin{aligned} P_t &= \left(\int_0^1 P_t(i)^{-\frac{1}{\mu}} di \right)^{-\mu} = \\ &= \left[(1-\theta) \left(P_t^{new} \right)^{-\frac{1}{\mu}} + \theta \left(P_{t-1} \right)^{\frac{-1}{\mu}} \right]^{-\mu} \end{aligned} \tag{18}$$

where the first equality follows from setting profits (10) to zero and substituting in (12).

Price setting with sticky prices: Log-linearised equation

- Dynamic AS curve (Phillips curve)

$$\theta\hat{\pi}_t = (1-\theta)(1-\beta\theta)\,\hat{mc}_t + \beta\theta E_t\hat{\pi}_{t+1} \tag{19}$$

Monetary policy

- Prices are sticky, so monetary policy has real effects
- Monetary authorities set the short-term (one period) nominal interest rate according to the Taylor-like feedback rule (see Taylor, 1993):

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\tilde{\pi}_t}\right)^{\gamma_\pi} \left(\frac{y_t}{\tilde{y}_t}\right)^{\gamma_y}\right]^{(1-\gamma_R)} e^{\varepsilon_{R,t}} \qquad (20)$$

  where:
  - $\tilde{\pi}_t$ and $\tilde{y}_t$ are target inflation and output (possibly time-varying) and R is the long-run equilibrium nominal interest rate (in practise: a model-dependent parameter)
  - γ_R – interest rate smoothing parameter, $\gamma_\pi > 1$, $\gamma_y \geq 0$.
  - $\varepsilon_{R,t}$ –iid monetary policy shock.
- Central bank can completely stabilise inflation by responding very aggressively to deviations of inflation from the target (i.e. by choosing a very large value for γ_π)

Log-linearised Taylor rule

$$\hat{R}_t = \gamma_R \hat{R}_{t-1} + (1 - \gamma_R)\left(\gamma_\pi \hat{\pi}_t + \gamma_y \hat{y}_t\right) + \varepsilon_{R,t} \tag{21}$$

Government

- Government role reduced as much as possible (possible extensions).
- Collects (Ricardian) taxes to finance exogenously given government expenditure

$$g_t = T_t \tag{22}$$

where g_t follows an AR(1) process (details later).

Market clearing conditions

- Output produced by firms must be equal to households' total spending (consumption):
$$c_t + g_t = y_t \tag{23}$$
- Labour supplied by households must be equal to labour demanded by firms, use (13) and (12):
$$l_t = \int_0^1 l_t(i)\,di = \int_0^1 \frac{y_t(i)}{z_t}di = \frac{\Delta_t y_t}{z_t} \tag{24}$$

where: $\Delta_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-(1+\mu)}{\mu}} di \geq 1$ is a measure of price dispersion ($\Delta_t = 1 \Leftrightarrow \forall i : P_t(i) = P_t$).
Notice that, similarly to (18) one can show that:
$$\Delta_t = (1-\theta)\left(\frac{P_t^{new}}{P_t}\right)^{\frac{-(1+\mu)}{\mu}} + \theta\left(\frac{P_{t-1}}{P_t}\right)^{\frac{-(1+\mu)}{\mu}}\Delta_{t-1} \tag{25}$$

Log-linearised market clearing conditions

- Output produced by firms must be equal to households' total spending (consumption):

$$\frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{g}}{\bar{y}}\hat{g}_t = \hat{y}_t \tag{26}$$

- Aggregate production function:

$$\hat{y}_t + \underbrace{\hat{\Delta}_t}_{=0} = \hat{z}_t + \hat{l}_t \tag{27}$$

Shocks

- AR(1) shocks

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_{g,t} \quad (28)$$

$$\hat{\psi}_t = \rho_\psi \hat{\psi}_{t-1} + \varepsilon_{\psi,t} \quad (29)$$

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z,t} \quad (30)$$

- Note monetary policy shock is *iid*.

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Simplification

- Assume that there is no government and $g_t = 0$.

Three equations governing the NK model

- From (26), (7), (15), (27), and (19) we get the following **New Keynesian Phillips (dynamic AS) curve**:

$$\theta\hat{\pi}_t = \beta\theta E_t\hat{\pi}_{t+1} + (1-\beta\theta)(1-\theta)(\varphi+\sigma)\hat{y}_t - (1-\beta\theta)(1-\theta)(1+\varphi)\hat{z}_t \quad (31)$$

- Log-linearised (26) and (8) imply the following **New Keynesian IS curve**:

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}\left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right) + \frac{1}{\sigma}\left(1-\rho_\psi\right)\hat{\psi}_t \quad (32)$$

- Log-linearised **Taylor Rule**, recall (21)

$$\hat{R}_t = \gamma_R\hat{R}_{t-1} + (1-\gamma_R)\left(\gamma_\pi\hat{\pi}_t + \gamma_y\hat{y}_t\right) + \varepsilon_{R,t}$$

Canonical NKM

- Demand and supply shocks ($\hat{\psi}_t$ and $\hat{z}_t$) are usually assumed to follow an AR(1) process.
- Monetary policy shock ($\varepsilon_{R,t}$) is usually assumed to be *iid* if interest rate smoothing motive is already taken into account
- Demand and monetary policy shocks move output gap and inflation in the same direction
- Supply shocks move output gap and inflation in the opposite directions, hence create trade-off between the two stabilisation objectives

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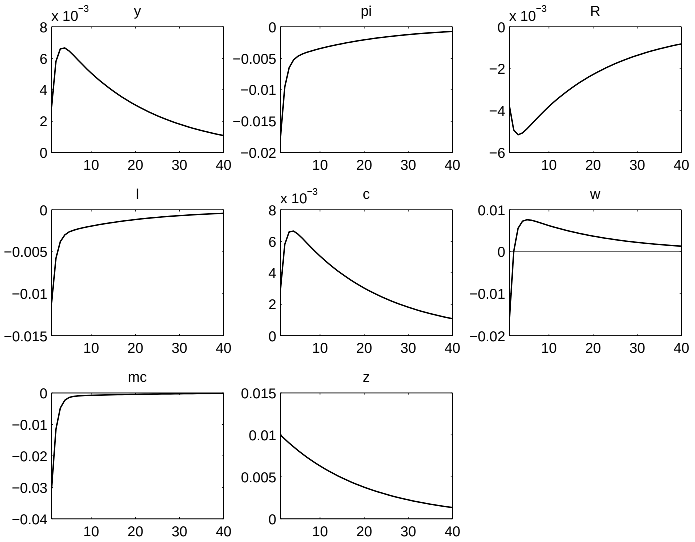
Parameters

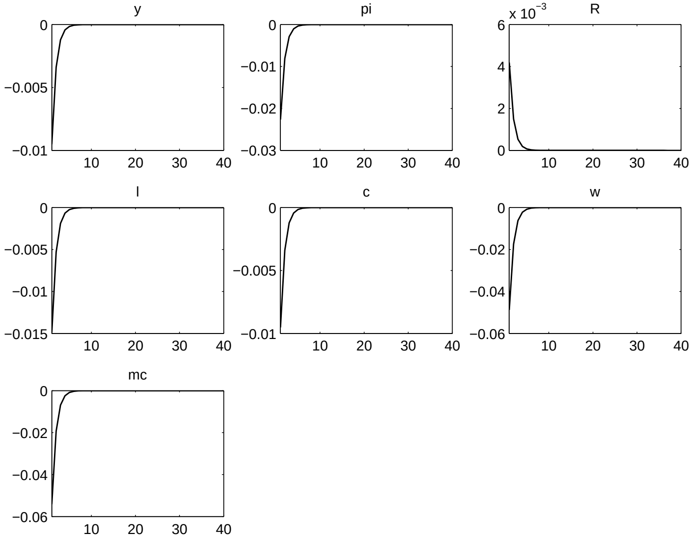
- Parameters used in numerical simulations:
  - $\sigma = 2$ (value for both CRRA and IES)
  - $\beta = 0.99$ (for quarterly data)
  - $\vartheta = 1$
  - $\varphi = 1$
  - $\mu = 0.2$ (implies a steady-state mark-up of 20%)
  - $\theta = 0.6$ (implies average time between price changes of 2.5 quarter)
  - $\gamma_R = 0.85$, $\gamma_\pi = 1.5$, $\gamma_y = 0.5$
  - $\rho_z = \rho_\psi = 0.95$, st. dev. (for all shocks) $= 0.01$.

- Note: ϑ and μ do not appear in the log-linearised version of the model.

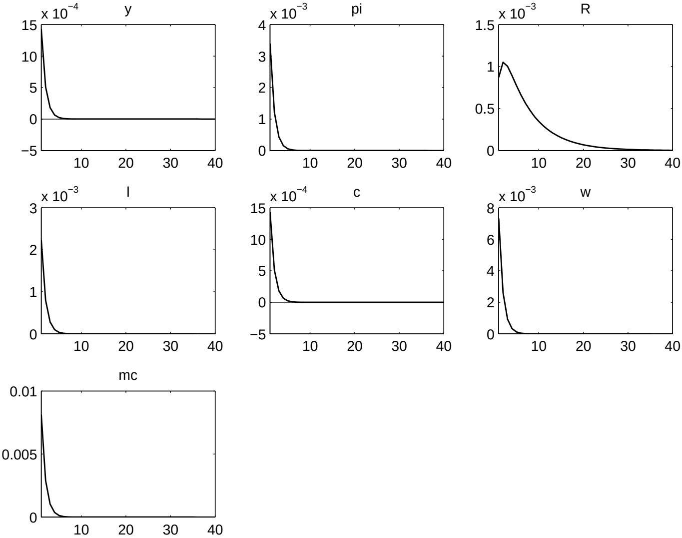
Technology shock



Monetary shock



Preference shock



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The role of expectations

- New Keynesian Philips curce—equation (31)—implies that current inflation is affected by inflation expectations
- Modern monetary policy: management of expectations
- Woodford (2005)[1] : *For not only do expectations about policy matter, (...) but very little else matters*

[1] Woodford, M. (2005), "Central-Bank Communication and Policy Effectiveness," paper presented at FRB Kansas City Symposium on "The Greenspan Era: Lessons for the Future," Jackson Hole, Wyoming, August 25-27, 2005.

The Taylor principle

- In the simple model if $\gamma_y = 0$ then it must be $\gamma_\pi > 1$ in order to have a unique equilibrium: the **Taylor principle**.
- This rules out self-fulfilling inflationary expectations.
- We neglect the zero bound problem.

Money?

- Our economy is cashless.
- Extending the model for money is quite simple. Put money in the utility function (with separability).
- This will give you the LM equation.
- But it will only allow to determine money supply necessary to keep the interest rate on the level necessary to support the nominal interest rate implied by the Taylor rule.

Optimal monetary policy

- Equation (24) implies that price dispersion (i.e. $\Delta_t > 1$) is costly
- Price dispersion can be eliminated if the central bank chooses to stabilise inflation at zero (i.e. sets the inflation target to zero and responds very aggressively to any deviations from the target)
- Hence, a policy strictly stabilising inflation can replicate the flexible price equilibrium
- However, monetary policy may face a trade-off between stabilising inflation and keeping output at a desired (not constant, in general) level
- This trade-off vanishes if ("divine coincidence"):
  - steady state output is efficient (i.e. distortions related to monopolistic competition are eliminated, e.g. by proper subsidies to firms)
  - there are no cost-push shocks (i.e. shocks to the Phillips curve)
- In this case perfect price stability is optimal.

Natural output and output gap

- Some models consider a Taylor rule that reacts to changes in output gap rather than deviations of GDP from the steady state:

$$\tilde{y}_t = y_t - y_t^n$$

where: y_t^n is natural (efficient) output, i.e. the level of output under flexible prices and no inefficient shocks (e.g. shocks to markups, distortionary taxes)

- In our case the natural level of output y_t^n is derived from the model by assuming $\theta = 0$ (flexible prices). It actually is quite simple.

- More generally: y_t^n will depend on all real and nondistortionary shocks in the economy (preference, government spending etc.)

Output gap version

- Clarida, Gali and Gertler (1999); Woodford (2003)
- 3 variables: output gap, inflation, nominal interest rate (all in log-deviations from the steady state)
- Simple manipulation allows to derive 3 log-linearised equations:
  - dynamic IS curve

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma}(R_t - E_t \pi_{t+1}) + \varepsilon_{d,t} \quad (33)$$

    where $\varepsilon_{d,t}$ depends on shocks in the model (in our model preference and productivity shocks)
  - Phillips curve (dynamic AS curve)

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \tilde{y}_t + \varepsilon_{s,t} \quad (34)$$

    where $\varepsilon_{s,t}$ collects supply shocks (e.g. shocks to mark-ups) and λ is a function of deep model parameters (γ, β, φ, θ)
  - Monetary policy rule (e.g. Taylor-like with smoothing)

$$R_t = \gamma_R R_{t-1} + (1 - \gamma_R)(\gamma_\pi \pi_t + \gamma_{\tilde{y}} \tilde{y}_t) + \varepsilon_{R,t} \quad (35)$$

Some implausible results

- No inflation persistence.
- No output persistence.

Existing extensions

- The simple model performs poorly. Thus needs extensions.
- Capital/investment adjustment costs - adds persistence, Tobin's q.
- Capital utilisation costs (introduces labour hoarding).
- Indexation - adds inflation persistence, (and hump shape in the monetary policy irf).
- Wage rigidities - adds persistence (including inflation persistence).
- Open economy.
- Financial markets - Bernanke, Gertler and Gilchrist (1999) or Iacoviello (2005).
- Search model of labour market - unemployment.

New Keynesian model - summary

- Very simple dynamic stochastic general equilibrium model (DSGE) with monopolistic competition and sticky prices
- Monopolistic power of firms $\implies$ decentralised allocations are not Pareto optimal (production not at an efficient level)
- Price stickiness restores the role of monetary policy:
  - Monetary policy has real effects (affects output, consumption, real wages)
  - The case for price stabilisation: price stability eliminates price dispersion
  - Pursuing strict price stabilisation is optimal if steady state distortions (due to monopolistic competition) are eliminated (e.g. by production subsidies)
- The workhorse model in central banks