

Homework 1: Solutions

Homework 1: Question 1

Consider an ARMA(1,2) model:

$$y_t = \phi y_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}, \quad \varepsilon_t \sim iidN(0, \sigma^2).$$

- (a) What are the stationarity conditions for this process?
- (b) What are the invertibility conditions for this process?
- (c) Express the process in the Wold form and specify Wold form coefficients in terms of ϕ and θ_i s.

Answer:

- (a) ARMA(1,2) process is stationary if and only if $|\phi| < 1$.
- (b) ARMA(1,2) is invertible when roots of characteristic equation $\theta(z) = 1 + \theta_1 z + \theta_2 z^2 = 0$ are outside the unit circle, i.e.

$$|z_{1,2}| = \left| \frac{-\theta_1 \pm \sqrt{\theta_1^2 - 4\theta_2}}{2\theta_2} \right| > 1$$

One can show that it corresponds to the following restrictions on θ_1 and θ_2 :

$$\theta_1 + \theta_2 > -1, \quad \theta_1 - \theta_2 < 1, \quad |\theta_2| < 1.$$

- (c) The Wold form in terms of ϕ and θ_i s, for $|\theta| < 1$, can be written as

$$y_t = \sum_{i=0}^{\infty} \psi_i \varepsilon_{t-i}$$

where

$$\begin{aligned} \psi_0 &= 1 \\ \psi_1 &= \phi + \theta_1 \\ \psi_i &= \phi^i + \theta_1 \phi^{i-1} + \theta_2 \phi^{i-2}, \quad \forall i \geq 2. \end{aligned}$$